STATISTICAL CORRELATION
OF THE
TIME RATE OF CHANGE OF
RELATIVE VORTICITY WITH
CONVECTIVE WEATHER PHENOMENA

BY

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PREFACE

This paper represents a statistical evaluation of the relationship between relative vorticity and convective weather phenomena. Specifically, it contains a correlation of both vorticity neglecting wind shear and individual rate of change of vorticity, with convective activity. Data for these determinations was obtained from surface maps covering a two and one-half year period, from January, 1949 to May, 1951.

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The author wishes to express gratitude for the assistance of Associate Professor G. J. Haltiner, Department of Aerology, U. S. Naval Postgraduate School, Monterey, California.

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TABLE OF SYMBOLS AND ABBREVIATIONS

C	Relative circulation
ΔA	Element of area
δA	Element of area
ΔC	Increment of relative circulation
$\Delta \mathbf{r}$	Increment of the position vector of a point on a curve
Δy	Northward component of the increment of distance
$\frac{\partial v}{\partial n}$	Wind shear along the normal to the flow
ζ	Scalar relative vorticity
ζ_a	Scalar absolute vorticity
∇_H	Two-dimensional grad
K_s	Curvature of a streamline
λ	Coriolis parameter
$\mathbf{r}$	Position vector of a point on a curve
$\mathbf{V}$	Wind velocity
v	Wind speed
v_y	Scalar northward component of wind speed
ϕ	Latitude
Ω	Angular velocity of the earth

I. INTRODUCTION

The source of inspiration for this investigation was the desire to test, on a statistical basis, the frequently-used forecasting rule that wind flow from the south causes convergence and resulting convective weather phenomena. The subject has been of interest to other investigators. Namias and Clapp, on 10,000-foot maps, computed under steady state conditions "inertia trajectories"; i.e., trajectories of parcels of air moving such that the vertical component of the absolute vorticity remained constant [5]. The inertia trajectory was determined by the initial latitude of the parcel, as well as curvature, lateral shear, speed, and direction of the parcel. These vorticity trajectories were found to be not parallel to the isobars, presumably due to the modifying effects of convergence or divergence. If the isobars curved more cyclonically than the trajectories, convergence was present; if more anti-cyclonically, divergence existed. The fields of divergence and convergence were then related to areas of heating and cooling. The results of this research were applied to the trajectory method of forecasting by making qualitative corrections to the computed vorticity trajectories on the five-day mean 10,000-foot charts and thereby obtaining a more accurate prognosis of the future positions of troughs and ridges at that level.

Another paper related to the subject appeared in 1946, in which Houghton and Austin constructed charts of horizontal divergence [4]. One possible approach to the problem of computing quantitatively the time rate of change of vorticity would involve the construction and use of such charts, but as will be seen later, a different technique has been employed.

A paper somewhat more closely allied to the subject under consideration was published in 1950 [1]. Relative vorticity was computed at the 700-millibar level by the use of the expression for vorticity in natural coordinates, and correlated with the heights of the tops of cumuliform clouds over the ocean along the Washington-Bermuda airways route. The authors had considerable cloud data. Pilots flying the route were required to construct cross-sections showing weather encountered along the route upon completion of each flight. A large initial sample of 454 observations was obtained, as well as 210 additional observations later. In the original sample, the linear coefficient of correlation between vorticity and cloud-top height was 0.84; in the later sample, 0.65.

In a paper that appeared in March, 1951, H. T. Harrison applied qualitative synoptic tests to a group of forecasting rules [2]. Pertinent to this discussion is the rule:

> A deep current of air moving northward across latitude lines in a straight or cyclonically curved path is undergoing convergence. Clouds and rainfall will develop which will be abundant if the airstream is curving cyclonically.

Harrison applied this rule to 39 actual cases, at the 700-millibar level, and found that 92% of them verified the rule.

The data in the above-mentioned papers was obtained from upper-level charts; in this paper, as will be explained later, mean sea level charts were used.

II. THE THEORY OF RELATIVE VORTICITY

By way of review recall that the definition of vorticity came about in connection with the subject of the relative circulation about closed individual curves in a fluid [3]. Relative vorticity (denoted by ζ) was defined as $\lim_{\Delta A \to 0} \frac{\Delta C}{\Delta A}$, and on the basis of this definition, the relative circulation about a closed curve C' was shown to be

$$C = \oint_{C'} \mathbf{V} \cdot \delta \mathbf{r} = \int_{S} \zeta \, \delta A$$

In rectangular coordinates, the vector representing relative vorticity is equal to the curl of relative velocity. In computing relative vorticity from actual weather maps, however, it was found more convenient to use the natural coordinate expression of this quantity:

$$\zeta = v K_s - \frac{\partial v}{\partial n} \qquad (1)$$

where:

v = wind speed

K_s = curvature of the streamlines (positive for cyclonic curvature; negative for anti-cyclonic curvature)

$\frac{\partial v}{\partial n}$ = wind shear along the normal to the flow. If the normal, n , is considered to the left of the flow, the quantity $\frac{\partial v}{\partial n}$ is negative for cyclonic shear, positive for anti-cyclonic.

The other basic equation pertinent to this development is the relative vorticity theorem [3]. The basic assumptions for this theorem were:

(1) Frictionless fluid,

(2) Nearly horizontal motions, and

(3) Horizontal solenoids negligible.

By the use of the relationship that the time rate of change of absolute vorticity per unit absolute vorticity is equal to the negative horizontal divergence of the velocity; i.e.,

$$\frac{1}{\zeta_a}\frac{d\zeta_a}{dt} = -\nabla_H \cdot V$$

The equation of relative vorticity was derived:

$$\frac{d\zeta}{dt} = -\frac{2\Omega\cos\phi\, v_y}{a} - (\zeta+\lambda)\nabla_H \cdot V \qquad (2)$$

where:

Ω = angular velocity of the earth (7.292×10^{-5} radians per second)

ϕ = latitude of the parcel of air

v_y = scalar northward component of the velocity of the wind

λ = coriolis parameter

$\nabla_H \cdot V$ = horizontal divergence of the velocity

Consider equation (2) for the case of southerly winds. If relative vorticity remains constant, $\frac{d\zeta}{dt} = 0$, and

$$\frac{2\Omega\cos\phi\, v_y}{a} = -(\zeta+\lambda)\nabla_H \cdot V \qquad (3)$$

For southerly flow, v_y is positive since the positive direction of y is taken as due north. Therefore, from equation (3), it is seen that horizontal convergence results. If vorticity is increasing, $\frac{d\zeta}{dt} > 0$, and from equation (2), horizontal convergence occurs. When vorticity is decreasing, $\frac{d\zeta}{dt} < 0$, which from equation (2) will occur either: (a) when the term $(\zeta+\lambda)\nabla_H \cdot V = 0$ (zero divergence); (b) when this same term is greater than zero (divergence); or (c), where this term is negative (convergence) but of smaller absolute value than $\frac{2\Omega\cos\phi\, v_y}{a}$.

Neglecting wind shear, these observations are easily depicted by a schematic diagram:

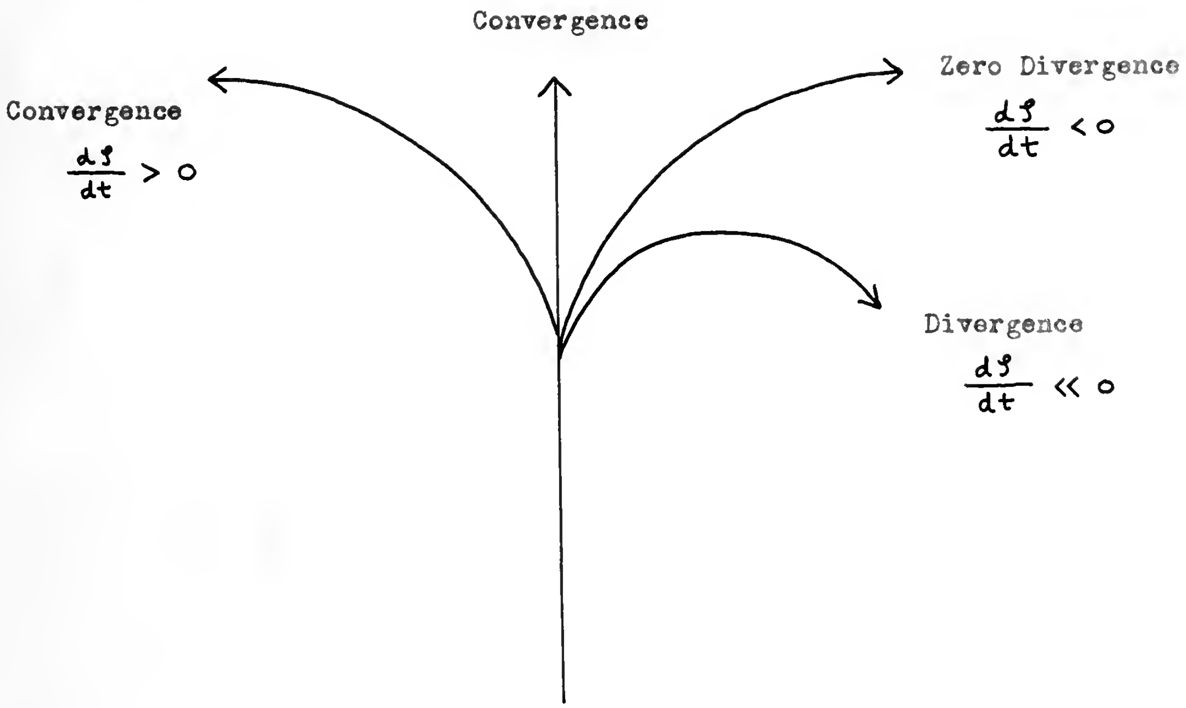

Figure 1.

Schematic Diagram Involving Southerly Winds

Now consider northerly flow, in which case v_y is negative. In straight northerly flow, it is apparent from equation (3) that divergence occurs. If vorticity is decreasing, $\frac{d\zeta}{dt} < 0$, and from equation (2), horizontal divergence results. $\frac{d\zeta}{dt} > 0$ either when: (a), the term $(\zeta + \lambda)\nabla_H \cdot V = 0$ (zero divergence); (b), this term is less than zero (convergence); or (c) this term is positive (divergence) but less in absolute value than $-\frac{2\Omega \cos\phi \, v_y}{a}$.

These remarks can be schematically represented, neglecting wind shear, as:

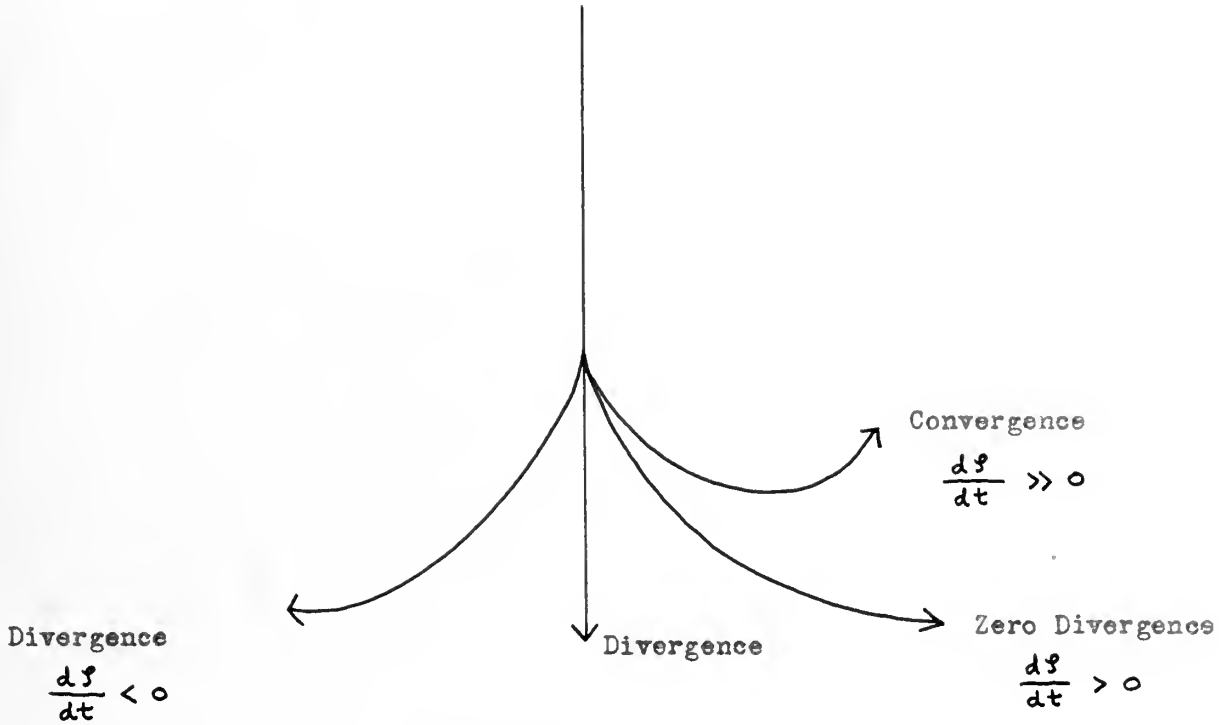

Figure 2.

Schematic Diagram Involving Northerly Winds

It has been tacitly assumed in the development above that the factor, $\zeta + \lambda$, is positive. That this is generally true can be easily shown. The coriolis parameter, λ , is always positive in the northern hemisphere. At latitude 30° it has the smallest magnitude in the geographical region considered in this paper, 0.26251 hours^{-1}. In 268 computations of ζ , only in two instances of negative vorticity did the absolute value exceed 0.26251.

The preceding observations may be summarized as follows:

(1) Convergence occurs when there is straight southerly flow; divergence, when straight northerly flow.

(2) When $\frac{d\zeta}{dt} > 0$; i.e., the flow is becoming less anti-cyclonic or more cyclonic, there is always convergence when the flow is southerly. There may be convergence or divergence when the flow is northerly, convergence occurring in cases where $\frac{d\zeta}{dt}$ is of a relatively larger order of magnitude.

(3) When $\frac{d\zeta}{dt} < 0$; i.e., the flow is becoming less cyclonic or more anti-cyclonic, there is always divergence when the flow is northerly; there may be divergence or convergence when the flow is southerly, divergence occurring when the absolute value of $\frac{d\zeta}{dt}$ is of a relatively larger order of magnitude.

Hence, it is apparent from theoretical considerations that the blanket use of the forecasting rules of prognosticating convergence and convective activity with southerly flow and divergence with northerly flow is invalid. However, one would be led to expect a fairly high positive correlation between the rate of change of vorticity and convective activity, providing of course, that the model on which the theorem is based is sufficiently representative of average atmospheric conditions.

III. THE CORRELATION OF RELATIVE VORTICITY NEGLECTING WIND SHEAR WITH CONVECTIVE WEATHER PHENOMENA

The author has collected observations from surface weather maps in order to obtain a measure of the extent to which the theory applies in the atmosphere. The first phase of the investigation involved only a correlation of relative vorticity itself with convective weather phenomena. One hundred and twenty eight calculations were made of relative vorticity in cases where wind shear was negligible, with the use of equation (1). To simplify the calculations, only situations approximating steady-state conditions were used. The isobars which were considered to be streamlines were then also regarded as approximate trajectories of the air parcels. The curvature of the streamlines, K_s, was determined by comparison with a series of arcs drawn on opaque paper. The units of curvature used were nautical miles^{-1}, and the velocity, v, estimated by use of a geostrophic wind scale, was measured in knots; i.e., nautical miles hours^{-1}. Thus the units of relative vorticity were hours^{-1}.

The average relative vorticity was computed for a square bounded by five degrees of latitude and five degrees of longitude. Such squares were considered only in the Great Plains region of the United States, generally from 30 to 55 degrees north latitude, and from 90 to 100 degrees west longitude. It was deemed inadvisable to consider areas where convection might be the result of orographic effects. Attention was also given to avoiding such cases where convection was the result of frontal lifting.

A "convective index" for the square area was then also computed. This was accomplished by assigning various weights to the different types of

clouds and other weather phenomena, higher weights being assigned to the weather types indicative of greater convective activity. Standard numerical symbols for clouds and weather types were used, as designated by the U. S. Weather Bureau [7]. The assigned weights are shown below:

Table 1.

Weight	N	C_L	C_M	C_H	WW
10		9			17 29 65 75 82 90-99
8		2,3,7	8		50-59 60-64,66-69 70-74,76-79 80-81,83-89
6					14-16 20-27
4		1,4,5,8	6,9		
2			3,4,5,7	9	
0	0	6	1,2	1-8	11 12 28 40-49

Table of Weights Assigned to Weather Phenomena

The convective index was computed by selecting from each reporting station in the square the symbol with the greatest weight and determining the average of these weights. The linear correlation coefficient was then obtained between the convective indices and the average relative vorticities, rounded off to the nearest hundreth and multiplied by 10^2 for the purpose of convenient units. Separate correlations were made for the cases of southerly winds, of northerly winds, and then of all cases combined. The scatter diagrams for these correlations and the results are shown on the succeeding pages.

TABLE 2. SCATTER DIAGRAM OF RELATIVE VORTICITY AND CONVECTIVE INDEX
(SOUTHERLY WINDS)

Convective Index

$\zeta \times 10^2$ Hours^{-1}

v_j \ u_i	-4	-3	-2	-1	0	1	2	3	4	5	Totals
-8											0
-7			2								2
-6											0
-5			1	1	1						3
-4	2		3								5
-3	2	3	5	3	1	3	1		2		20
-2	5	4	3	6	6	3	3		1	1	32
-1	5	1	3	1	2	1	1	1			15
0	3		4	2	2		1				12
1				1	1						2
2					1						1
3											0
Totals	17	8	21	14	14	7	6	1	3	1	92

TABLE 3. SCATTER DIAGRAM OF RELATIVE VORTICITY AND CONVECTIVE INDEX
(NORTHERLY WINDS)

Convective Index

$\zeta \times 10^2$ Hours^{-1}

v_j \ u_i	-4	-3	-2	-1	0	1	2	3	4	5	Totals
-16					1						1
-15											0
-14											0
-13											0
-12											0
-11											0
-10											0
-9											0
-8											0
-7											0
-6											0
-5		2					1				3
-4	2				1	2					5
-3	3	1									4
-2	3	2	1	1							7
-1	2	1									3
0	1	2				1		1	2		7
1							1				1
2		1							1		2
3							1				1
4									1		1
5											0
6											0
7											0
8											0
9									1		1
Totals	11	9	1	1	2	3	3	1	5	0	36

TABLE 4. SCATTER DIAGRAM OF RELATIVE VORTICITY AND CONVECTIVE INDEX
(COMBINED SOUTHERLY AND NORTHERLY WINDS)

Convective Index

$\zeta \times 10^2$ Hours^{-1}

v_j \ u_i	-4	-3	-2	-1	0	1	2	3	4	5	Totals
-16					1						1
-15											0
-14											0
-13											0
-12											0
-11											0
-10											0
-9											0
-8											0
-7			2								2
-6											0
-5		2	1	1	1		1				6
-4	4		3		1	2					10
-3	5	4	5	3	1	3	1		2		24
-2	8	6	4	7	6	3	3		1	1	39
-1	7	2	3	1	2	1	1	1			18
0	4	2	4	2	2	1	1	1	2		19
1				1	1		1				3
2		1			1				1		3
3							1				1
4									1		1
5											0
6											0
7											0
8											0
9									1		1
Totals	28	17	22	15	16	10	9	2	8	1	128

Table 5.

Wind Flow	Average Vorticity	Range of Vorticity	Average Convective Index	Range of Convective Index	r
Southerly Winds	-.02005	-.12987 to .05560	3.310	.00 to 9.33	.0232
Northerly Winds	-.01283	-.30000 to .20850	3.328	.00 to 8.00	.3408
Combined	-.01802	-.30000 to .20850	3.315	.00 to 9.33	.2744

Results of Correlation of Vorticity and Convective Index

Another paper has been prepared in conjunction with this study, correlating convection with relative vorticity where wind shear was included [6]. In his research, Lieutenant Tatone made 140 computations of convective indices and relative vorticity. The same measuring techniques were employed, and a linear correlation coefficient of -0.1050 for all cases combined was obtained.

From these computations it is seen that vorticity itself did not correlate well with convective activity. In the Washington-Bermuda airways study, better correlations were obtained; 0.84 in the sample of 454 observations and 0.65 in the sample of 210 [1]. A possible explanation for these higher correlations, aside from the fact that the samples were larger, is that vorticity at the 700-millibar level is perhaps more representative for the layer above the earth's surface than vorticity at the surface itself. Also, the height of cloud tops might be a better indication of convective activity than surface weather observations.

IV. THE CORRELATION OF THE TIME RATE OF CHANGE OF RELATIVE VORTICITY WITH CONVECTIVE WEATHER PHENOMENA

The final phase of this investigation involved the correlation of the time rate of change of relative vorticity with convective weather phenomena, under nearly steady-state conditions. By dividing equation (2) by v_y is obtained the relationship:

$$\frac{d\zeta}{v_y\,dt} = \frac{d\zeta}{dy} = -\frac{2\Omega\cos\phi}{a} - \frac{\zeta+\lambda}{v_y}\,\nabla_H \cdot V$$

The quantity, $\frac{d\zeta}{dy}$, was computed by the use of finite differences: $\frac{\Delta\zeta}{\Delta y} = \frac{\zeta_2 - \zeta_1}{\Delta y}$. Two adjacent square areas were selected such that the air parcel could be considered as moving along the streamlines from the center of the first square to the center of the second. At the center position of each square, the parcel was regarded as possessing the average relative vorticity for that square. Since the squares were adjacent and five degrees of latitude in size, the movement of the parcel, Δy, was 300 nautical miles, positive for southerly winds, negative for northerly winds.

Vorticity was determined again by the use of equation (1); however, in this phase of the work wind shear was included. In measuring the shear, the direction of the normal was regarded to the left of the direction of flow. The term $\frac{\partial v}{\partial n}$, or $\frac{\Delta v}{\Delta n}$, was then negative when the flow decreased along the normal (cyclonic shear), positive when it increased (anti-cyclonic shear). Then the two terms of equation (1) were added algebraically to obtain ζ, and the rate of change of ζ determined from the relationship: $\frac{\Delta\zeta}{\Delta y} = \frac{\zeta_2 - \zeta_1}{\Delta y}$, in hours^{-1} nautical miles^{-1}.

The convective index for each square was computed according to the method described previously; i.e., of calculating the average of the highest-weighted phenomena reported by the stations in that area. Then the indices of the two squares were used to arrive at an average convective index associated with the movement of the parcel. The same geographical area was used and again instances where frontal activity was present were avoided.

For convenience in units, the scatter diagram was based on $\frac{d\zeta}{dy} \times 10^{4}$. It is shown below. Since only 9 of the 70 observations involved northerly winds, all cases were combined on a single scatter diagram.

A coefficient of 0.0513 was obtained, representing the linear correlation between the convective index and the rate of change of relative vorticity. The average convective index was 2.952 and the average rate was 0.0000424 hours^{-1} nautical miles^{-1}.

TABLE 6. SCATTER DIAGRAM OF RATE OF CHANGE OF RELATIVE VORTICITY AND CONVECTIVE INDEX

Convective Index

v_i \ u_i	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	Totals
-17			1															1
-16																		0
-15																		0
-14																		0
-13								1										1
-12	2																	2
-11							1				1							2
-10																		0
-9	1								1									2
-8						1												1
-7			1															1

TABLE 6. SCATTER DIAGRAM OF RATE OF CHANGE OF RELATIVE VORTICITY AND CONVECTIVE INDEX

Convective Index

	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	Totals
-17			1															1
-16																		0
-15																		0
-14																		[illegible]
-13								1										1
-12	[illegible]																	[illegible]
-11							1				1							2
-10																		0
-9	1								1									[illegible]
-8						[illegible]												1
-7			[illegible]															1
-6				1		[illegible]				[illegible]								[illegible]
-5									1						1			[illegible]
-4			[illegible]											1				[illegible]
-3	[illegible]																	[illegible]
-2			[illegible]			1	[illegible]	[illegible]	[illegible]				1			1		5
-1	[illegible]		[illegible]		1			1	[illegible]					1			1	[illegible]
0			1				1				1							3
1	[illegible]				1													3
2						1		1		1								3
3	3					1		1	2					1				8
4		1		2	2	1	1		1									5
5	1		1				1	1										4
6																		0
7																		0
8																		0
9																		0
10																		0
11																		0
12		1		1							1							3
13																		0
14																		0
15																		0
16																		0
17																		0
18																		0
19												1						1
	11	[illegible]	10	4	4	6	[illegible]	[illegible]	[illegible]	[illegible]	3	1	1	[illegible]	1	1	1	70

V. RESULTS AND INTERPRETATIONS

On the basis of the observations and measurements made in this investigation it appears that under conditions approximating those of steady-state neither vorticity nor the rate of change of relative vorticity correlates well with convective activity. One pertinent factor, not considered, is the moisture content of the air. However, since the wind flow in the majority of the cases was from the south, and had a long trajectory over the Gulf of Mexico, the moisture present was assumed to be adequate for the production of weather, given sufficient convection. It may also be true that the layer of air as represented by sea level isobars, extending from the surface to about 5,000 feet, is not sufficiently deep for testing the theory. Perhaps the layer from the surface to 500 or 700 millibars would give a better correlation. It is also possible that the assumptions on which the relative vorticity theorem is based are too restricted and do not represent average atmospheric conditions. Finally, the measuring techniques may have been too crude; however, the quantities that were involved in this work were measured as accurately as possible from maps available in the normal aerological office.

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